L44 GENERATION AND RECOMBINATION

- *Generation-Recombination mechanisms*

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} + G - \frac{\delta n}{\tau}$$

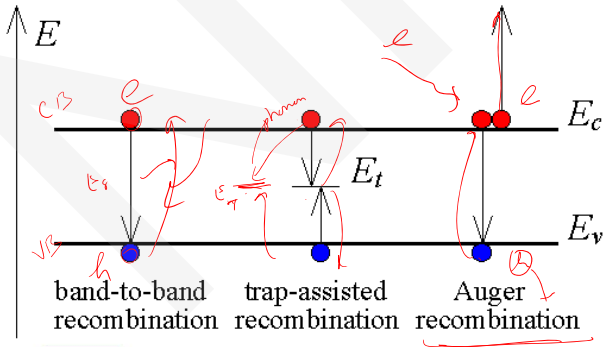
$$\frac{dp}{dt} = -\frac{1}{q}\frac{dJ_p}{dx} + G - \frac{\delta p}{\tau}$$

GENERATION-RECOMBINATION (GR)

- At equilibrium charge distributions: $np = n_i^2$
- When the system is perturbed, the system tries to restore itself towards equilibrium through recombination-generation
- Steady-state recombination rate will be proportional to the deviation from equilibrium,
  - $R \propto (np - n_i^2)$

GR MECHANISMS

- Recombination: process by which electrons and holes annihilate each other i.e. : electrons occupy - through one or multiple steps - the empty state associated with a hole.
- Energy released as- photon / phonon / kinetic energy of a carrier



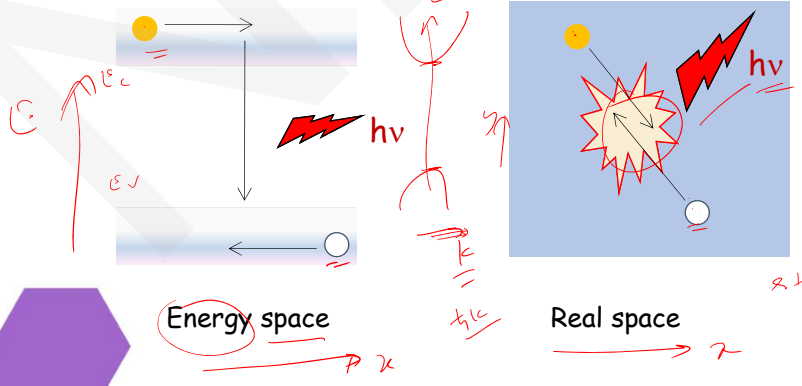
- These recombination mechanisms are reversible, i.e., carrier generation.
- A single expression can be used to describe generation-recombination

GR MECHANISMS

- Two particle
  - Energy-level consideration
    - One step (Direct)
      - Photogeneration
      - Radiative recombination
      - Direct thermal generation
      - Direct thermal recomb.
    - Two-step (indirect)
      - Shockley-Read-Hall (SRH) generation-recombination
      - Surface generation-recombination
- Three particle
  - Impact ionization → Pure generation process
  - Auger
    - Electron emission
    - Hole emission
    - Electron capture
    - Hole capture

GR MECHANISMS

1. Band-to-Band Recombination, i.e. direct thermal recombination, involves the electron falling from an allowed conduction band state into a vacant valence band state. This process is typically radiative. Eg Lasers, LEDs



$$R = \alpha_r (np - n_i^2) = \frac{\delta n}{\tau}$$

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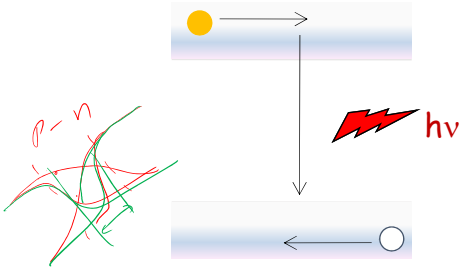


$$LLI - n_0 << \Delta n = \Delta p << p_0 = N_A$$

$$\frac{1}{\tau} = \alpha_r N_A$$

$$HLI - \delta = \Delta n = \Delta p >> p_0, n_0$$

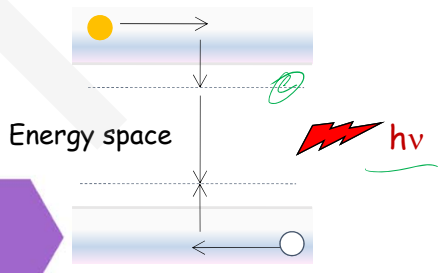
$$\frac{1}{\tau} = \alpha_r \delta$$

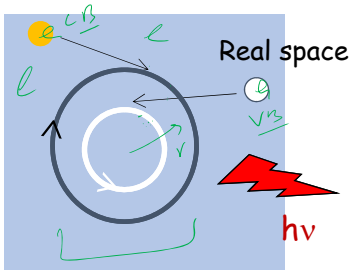
$$Depletion - n \sim p \sim 0$$

$$R = -\alpha_r n_i^2$$

GR MECHANISMS

2. Recombination Involving Excitons (Coupled electron-hole pair, which moves as a unit in response to applied forces): Exciton introduces a temporary level into the band gap slightly above the valence band edge or slightly below the conduction band edge. It give rise to subband-gap radiation, usually at low temperatures and in Light Emitting Diodes (LEDs) containing shallow-level isoelectronic centers, Organic Solar cells, CNTs, wires (1-D systems)

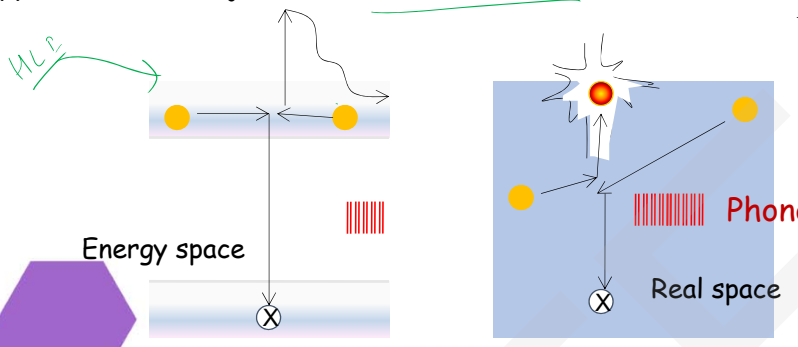




GR MECHANISMS

3. Auger Recombination: a non-radiative process, where band-to-band recombination or trapping at a band gap center occurs simultaneously with the collision between two like carriers. The surviving carrier takes away the energy and thermally loses it. Dominates in highly doped III-Vs, concentrator type solar cells, junction lasers, and LEDs.

$$R = (np - n_i^2)(nC_n + pC_p)$$



$C_n, C_p \sim 10^{-30}\ cm^6/s$ at $300K$

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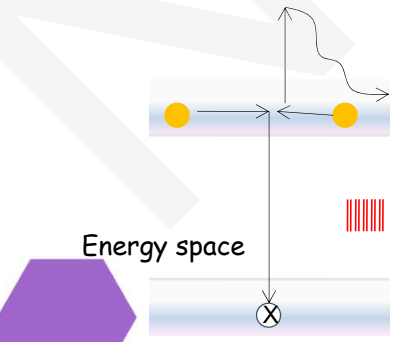


$$LLI - n_0 << \Delta n = \Delta p << p_0 = N_A$$

$$\frac{1}{\tau} = C_n N_A^2$$

$$HLI - \delta = \Delta n = \Delta p >> p_0, n_0$$

$$\frac{1}{\tau} = (C_n + C_p)\delta^2$$

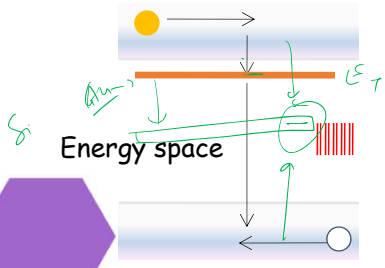
$C_n, C_p \sim 10^{-30}\ cm^6/s$ at $300K$

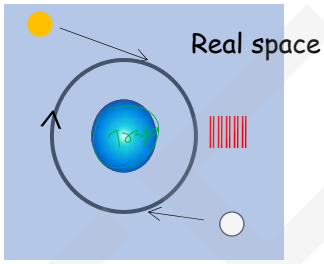
SEMICONDUCTOR DEVICE MODELING AND SIMULATION

GR MECHANISMS

4. R-G Center (indirect thermal) Recombination: R-G centers, energy levels (E_T) introduced by certain impurity atoms or crystal defects into the midgap region of a semiconductor, act as intermediaries in the recombination process. The capture of an electron and a hole at the same site leads to the annihilation of the electron-hole pair. This process is characteristically non-radiative.

$$R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$





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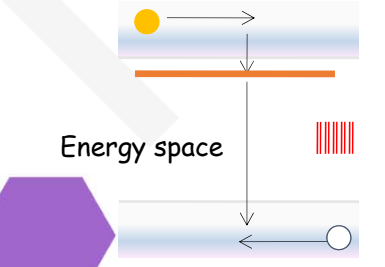


$$LLI - n_0 << \Delta n = \Delta p << p_0 = N_A$$

$$\tau = \tau_n$$

$$HLI - \delta = \Delta n = \Delta p >> p_0, n_0$$

$$\tau = \tau_n + \tau_p$$

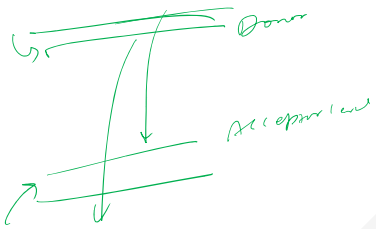
$$Depletion - n \sim p \sim 0$$

$$R = \frac{-n_i^2}{\tau_p(n_1) + \tau_n(p_1)}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

GR MECHANISMS

5. Recombination via Shallow Levels: Donor and acceptor sites function R-G centers, extremely inefficient at room temperature, in the recombination process. The largest energy step is typically radiative.



CONCLUSION

- Discussed various recombination processes

NPTEL

L45 GENERATION AND RECOMBINATION

- *Generation-Recombination mechanisms - derivation and measurements*

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} + G - \frac{\delta n}{\tau}$$

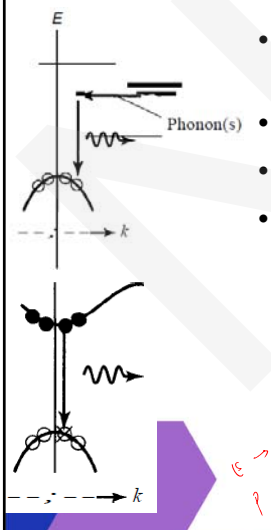
$$\frac{dp}{dt} = -\frac{1}{q}\frac{dJ_p}{dx} + G - \frac{\delta p}{\tau}$$

LUMINESCENCE

Luminescence - process of emitting light When electron-hole pairs are generated or carriers are excited into higher energy levels from which they fall to their equilibrium states.

- Photoluminescence -carriers excited by photon absorption
  - fluorescence-fast luminescent processes (10^{-8} sec)
  - phosphorescence-slow (second or minutes) luminescent processes.
- Cathodoluminescence - carriers created by high-energy electron bombardment
- Electroluminescence - excitation occurs by electric current (eg LED)

PROPERTIES OF GR PROCESS



- crystal momentum and energy is conserved
- Band-to-band recombination is much faster in direct gap semiconductors
- R-G center process is important in indirect gap semiconductors
- Usually only dominant process matters
- band-to-band or R-G center recombination-generation usually dominant in low-field regions of a non-degenerately doped semiconductor maintained at room temperature.

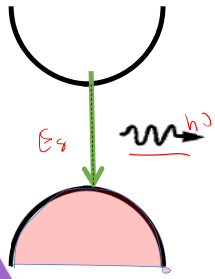
MOMENTUM CONSIDERATION

Conserve Energy $E_g = h\nu$

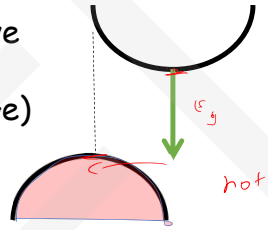
Conserve momentum $\Delta k = k_{photon} = 2\pi\nu/c = E_g/\hbar c$

$\Delta k \sim 6/\mu m$ ($E_g \sim 1.2$ eV) $\ll$ BZ $= 2\pi/a \sim 1/$ Å for a=5.43 Å Optical transitions almost vertical !!

Direct bandgap



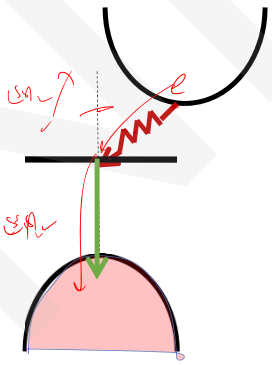
photons making vertical transitions won't conserve momentum for indirect band-gaps (Si, Ge)



Indirect bandgap

MOMENTUM CONSIDERATION

Indirect Band-gap



Two step process !!

$k_{trap} \approx 2\pi/a$

Conserve momentum

$\Delta k = k_{phonon} = 2\pi\nu_{phonon}/v_{sound}$

$= E_{phonon}/\hbar v_{sound} \approx 2\pi/a$ [Å]

Atomic displacements are involved, Wavelengths are Comparable

Conserve Energy

$E_{phonon} = h\nu_{phonon} \sim$ 10-100 meV $\ll E_g$

Eg for Si,

$v_L = 8430$ m/s along [100]

$\nu_{LTO}(\Gamma_{25'}) = 15.5\times10^{12}$ Hz T=300K

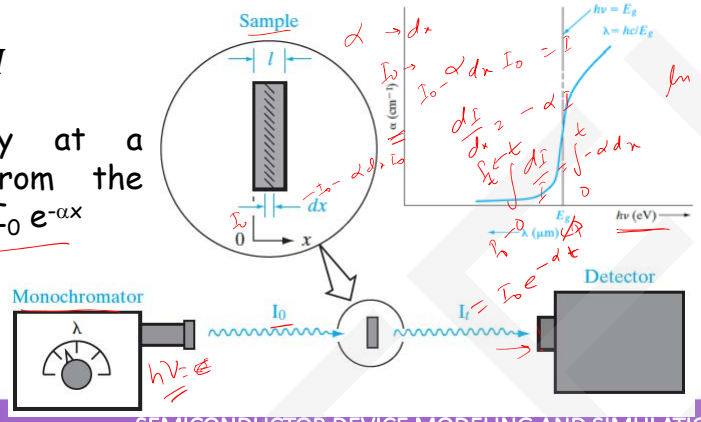
http://www.ioffe.ru/SVA/NSM/Semicond/Si/mechanic.html

OPTICAL ABSORPTION

- Relative transmission of the photons is observed. *Used to* measure band gap energy
- I_0 = photon beam of intensity (photons/cm^2-s)
- λ = photons of wavelength, selected by a monochromator
- l = sample of thickness, α=absorption coefficient (cm^{-1})

$$\frac{dI}{dx} = -\alpha I$$

Beam intensity at a distance x from the surface, $I(x) = I_0 e^{-\alpha x}$



PHOTOCONDUCTIVITY

- Excess electrons and holes creation ➔ conductivity increases
- photoconductivity – if excess carriers due to optical luminescence
- every semiconductor device depends in some way on the recombination of excess electrons and holes

Let us examine mechanisms by which excess electrons and holes recombine

DIRECT RECOMBINATION

- Excess electrons and holes decay by electrons falling from CB to empty states (holes) in VB. (photon emission)
- Spontaneous process i.e. constant probability that an electron and a hole will recombine, α_r
- rate of decay of electrons at any time t,

Thermal generation | Recombination

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)p(t)$$

- $\delta n(t)$, $\delta p(t)$ = instantaneous excess carrier concentrations

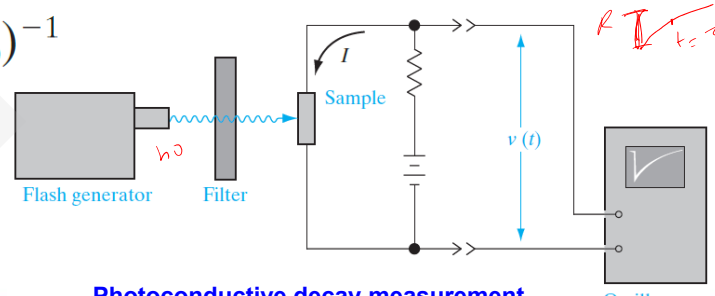
$$\frac{d\delta n(t)}{dt} = \alpha_r n_i^2 - \alpha_r[n_0 + \delta n(t)][p_0 + \delta p(t)]$$

$$= -\alpha_r[(n_0 + p_0)\delta n(t) + \delta n^2(t)]$$

DIRECT RECOMBINATION

- Case: Low level injection (δn (t=0) << p_0 for p-type)
- Equation simplifies to $\frac{d\delta n(t)}{dt} = -\alpha_r p_0 \delta n(t)$
- Solution, $\delta n(t) = \Delta n e^{-\alpha_r p_0 t} = \Delta n e^{-t/\tau_n}$
- Minority carrier lifetime,

$$\tau_n = (\alpha_r p_0)^{-1}$$



Photoconductive decay measurement

QUASI FERMI-LEVEL

- At thermal equilibrium $g(T) = \alpha_r n_i^2 = \alpha_r n_0 p_0$
- Light turned on (optical generation)

$$g(T) + g_{op} = \alpha_r np = \alpha_r (n_0 + \delta n)(p_0 + \delta p)$$

- For low level excitation, $g_{op} = \alpha_r (n_0 + p_0)\delta n = \frac{\delta n}{\tau_n}$
- Excess carriers, $\delta n = \delta p = g_{op}\tau_n$

$$\delta n = \tau_n g_{op} \quad \text{and} \quad \delta p = \tau_p g_{op}$$

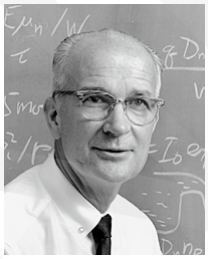
general expression (with trapping)

- Calc E_{fn}, E_{Fp}

$$n = n_i e^{(F_n - E_i)/kT}$$

$$p = n_i e^{(E_i - F_p)/kT}$$

INDIRECT RECOMBINATION



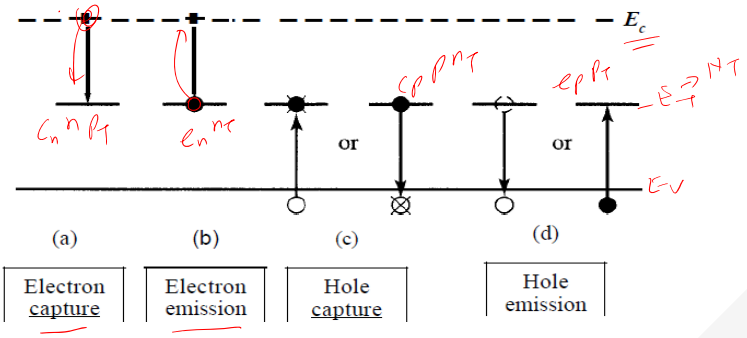
William Shockley

Robert N. Hall

Also called
Shockley-Read-Hall (SRH) Recombination

GR STATISTICS

Electronic transitions between a single-level R-G center and energy bands



$$\left.\frac{\partial n}{\partial t}\right|_{R-G} = \left.\frac{\partial n}{\partial t}\right|_{(a)} + \left.\frac{\partial n}{\partial t}\right|_{(b)}$$

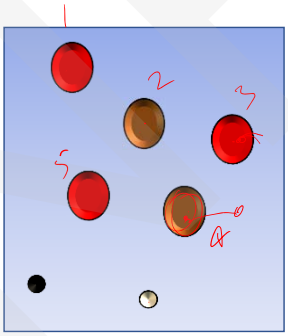
$$\left.\frac{\partial p}{\partial t}\right|_{R-G} = \left.\frac{\partial p}{\partial t}\right|_{(c)} + \left.\frac{\partial p}{\partial t}\right|_{(d)}$$

$$r_N = -\frac{\partial n}{\partial t} = c_n n p_T - e_n n_T$$

$$r_P = -\frac{\partial p}{\partial t} = c_p p n_T - e_p p_T$$

- n_T = number of filled (with electron) R-G centers cm^{-3}
- p_T = number of empty R-G centers cm^{-3}
- N_T = Total number of R-G centers cm^{-3}
- c_n = the electron capture coefficient(cm^3/sec)
- e_n = the electron emission coefficient (1/sec)

CAPTURE PROCESS



$$N_T = n_T + p_T$$

5 3 2

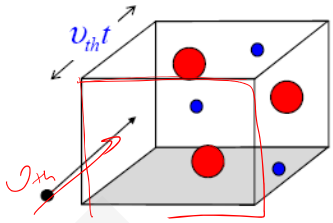
CAPTURE RATE

dn/dt = # of electron in small volume × relative cross section of empty traps / time

Thermal velocity, $v_{th} = \sqrt{3kT/m^*} \sim 10^7 \frac{\text{cm}}{\text{s}}$ at 300 K

Volume = $Av_{th}t$,
fractional area for capture = $p_T A\sigma_n v_{th} t/A$

prob of capture/sec = $p_T\sigma_n v_{th} = c_n p_T$



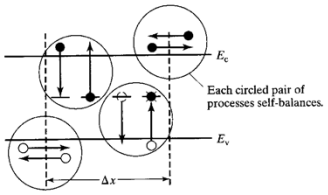
$$\frac{dn}{dt} = -n \times \left[\frac{A \times v_{th}t \times p_T \times \sigma_n}{A \times t}\right] \equiv -c_n p_T n \qquad c_n \equiv \sigma_n v_{th}$$

Assumption: these c's don't change under bias

GR STATISTICS

- ***Under equilibrium:***
  - self balancing of each fundamental process
  - $r_N = 0$ and $r_P = 0$
  - $e_{n0} = c_{n0} p_{T0} n_0 / n_{T0} = c_{n0} n_1$
  - $e_{p0} = c_{p0} n_{T0} p_0 / p_{T0} = c_{p0} p_1$

$$n_1 = \frac{p_{T0} n_0}{n_{T0}} = \left(\frac{N_T - n_{T0}}{n_{T0}}\right) n_0 = \left(\frac{N_T}{n_{T0}} - 1\right) n_0$$



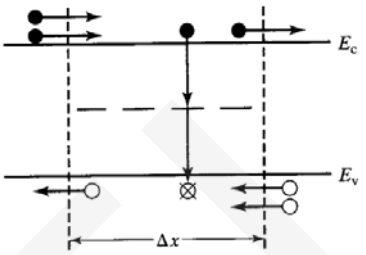
- Assuming emission and capture coefficient remain approximately equal to their equilibrium value
  - $e_n \simeq e_{n0} = c_{n0} n_1 \simeq c_n n_1$
  - $e_p \simeq e_{p0} = c_{p0} p_1 \simeq c_p p_1$
  - $n_0 = n_i \exp(E_F - E_i)/kT$

$$r_N = -\frac{\partial n}{\partial t} = c_n(n p_T - n_1 n_T)$$
$$r_P = -\frac{\partial p}{\partial t} = c_p(p n_T - p_1 p_T)$$

- For nondegenerate semiconductor
  - $n_{T0}/N_T = 1/(1+\exp(E_T' - E_F)/kT)$, $E_T' = E_T \pm kT \ln g_T$ and g_T is degeneracy factor
  - $n_1 = n_i \exp((E_T' - E_i)/kT)$ and $p_1 = n_i \exp((E_i - E_T')/kT)$

GR STATISTICS

- ***Under steady state:*** status quo is maintained by trade-off b/w processes (Assume emission and capture coefficient ≃ equilibrium value)
  - $r_N = r_P$
- $n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n(n+n_1)+c_p(p+p_1)}$
- $R \equiv r_N \equiv r_P = \frac{np - n_i^2}{\tau_p (n+n_1)+\tau_n(p+p_1)}$
- $\tau_p = \frac{1}{c_p N_T}$ and $\tau_n = \frac{1}{c_n N_T}$ are hole and electron minority carrier lifetimes



GR STATISTICS – TYPICAL VALUES

Material	N_T [cm^{-3}]	E_T [eV]	σ_N [m^2]	σ_P [m^2]	S_N [m/s]	S_P [m/s]
Si, Ge	1e13	0.0	1e-15	1e-15	0.0	0.0
III-Vs	2e16	0.4	1e-14	1e-13	0.0	0.0

MINORITY CARRIER LIFETIME

- The R-G centers are typically impurities that were unintentionally incorporated into the lattice during growth or processing
- Lifetime is decreased by adding impurities (Au to Si)
- Lifetime is increased by gettering impurities
  - diffuse phosphorous into the back side of an Si device in order to trap impurities and increase the lifetimes

Table 5.1 Observed Carrier Lifetimes in Si (300 °K).

Lifetime Range	Result-Reference	Device Structure
10^{-4}–10^{-2} sec	τ_0 = 2 msec [10]	Gettered Metal/SiO_2/Si (MOS) capacitors
10^{-6}–10^{-4} sec	τ_n = 23.5 μsec, τ_p = 1.5 μsec [11]	*pn*-junction devices
10^{-8}–10^{-6} sec	τ_n = 0.75 μsec, τ_p = 0.25 μsec [11]	Au-diffused *pn*-junction devices

LOW LEVEL INJECTION

- Low level injection:
  - changes in the carrier concentrations are much less than the majority carrier concentration under equilibrium conditions
  - $\Delta n = n - n_o$ and $\Delta p = p - p_o$
  - $\Delta n, \Delta p \ll n_o$ (n-type material), $\Delta n, \Delta p \ll p_o$ (p-type material)
- Assumptions about the R-G center parameters
  - R-G centers introduce E_T' level close to midgap ➔ $n_1 \sim p_1 \sim n_i$,
  - R-G center concentration is sufficiently low ➔ $\Delta n \sim \Delta p$
  - τ_n and τ_p are taken to differ by no more than a few orders of magnitude
- Low level injection
  - $R = \Delta n/\tau_n$ (p-type); $R = \Delta p/\tau_p$ (n-type);
- High level injection
  - $R = \Delta p/(\tau_p + \tau_n)$ where $\Delta n, \Delta p \gg n_o$ or p_o

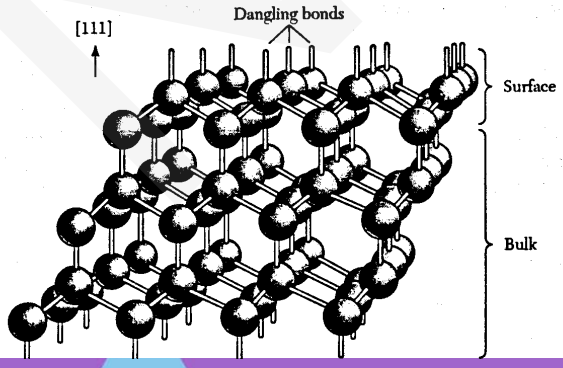
$$R = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)}$$

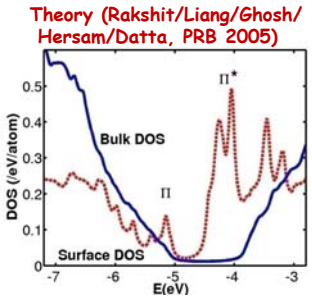
RG DEPLTION AND CAPTURE CROSS-SECTION

- Electrostatic depletion region:
  - n , p << $|N_D-N_A|$, does not imply carrier deficit
  - System in equilibrium ➔ n= n_o, p = p_o and np=n_i^2
- R-G depletion defined as :
  - semiconductor volume where n « n_l and p « p_1
  - np « $n_l p_1 = n_i^2$, deficit of carriers
  - R= $\frac{-n_i^2}{\tau_p n_1 + \tau_n p_1} = -\frac{n_i}{\tau_0}$ where $\tau_0=\tau_n+\tau_p$ *if* $E_T' = Ei$

SURFACE RECOMBINATION

- Lattice periodicity broken at surface/interface – mid-gap E levels
- Carriers generated-recombined per unit area
- Processes and descriptions analogous to bulk R-G using surface parameters





SURFACE RECOMBINATION

- For a ***single energy level*** surface state:

$$r_N = c_n(p_T n - n_T n_1) \qquad r_{Ns} = c_{ns}(p_{Ts} n_s - n_{Ts} n_{1s})$$
$$r_P = c_p(n_T p - p_T p_1) \qquad r_{Ps} = c_{ps}(n_{sT} p_s - p_{Ts} p_{1s})$$

$$n_1 = n_i e^{(E_T' - E_i)/kT} \qquad n_{1s} = n_i e^{(E_{IT}' - E_i)/kT}$$
$$p_1 = p_i e^{(E_i - E_T')/kT} \qquad p_{1s} = p_i e^{(E_i - E_{IT}')/kT}$$

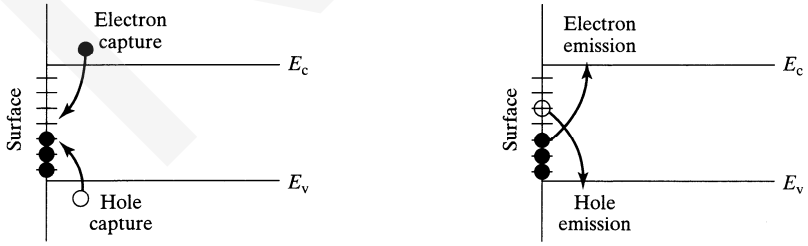
$$R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \qquad R = \frac{n_s p_s - n_i^2}{\frac{1}{s_p}(n_s + n_{1s}) + \frac{1}{s_n}(p_s + p_{1s})}$$

Surface recombination velocities $s_p \equiv c_{ps} N_{Ts}$ *and* $s_n \equiv c_{ns} N_{Ts}$

MULTI-LEVEL (MORE REALISTIC)

$D_{IT}(E)$ – density of interface traps (per unit area-energy)
$D_{IT}(E)dE$ – density of IT between E and E+dE (replaces N_{Ts})

$$R = \int_{E_v}^{E_c} \frac{n_s p_s - n_i^2}{\frac{1}{c_{ps}}(n_s + n_{1s}) + \frac{1}{c_{ns}}(p_s + p_{1s})} D_{IT}(E)dE$$



(a) Capture (b) Emission

CONCLUSION

- Discussed optical absorption, surface recombination and derived SRH recombination

Thank you

L46 RECALL DD EQUATIONS

Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{d}{dx}\left(\frac{\varepsilon d\psi}{dx}\right) = -\rho = -q\left(p - n + N_D^+ - N_A^-\right)$$

Carrier density equations

$$n = n_i.\exp\left(\frac{E_F - E_I}{k_B T}\right) = n_i.\exp\left(\frac{q.(\psi - \varphi_n)}{k_B T}\right) = n_i.\exp\left(\frac{V}{V_T}\right)$$

$$p = n_i.\exp\left(\frac{E_I - E_F}{k_B T}\right) = n_i.\exp\left(\frac{q.(\varphi_p - \psi)}{k_B T}\right) = n_i.\exp\left(-\frac{V}{V_T}\right)$$

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

current density equations

$$\vec{J}_n = -qn\mu_n\frac{\partial \psi}{\partial x} + qD_n\frac{\partial n}{\partial x}$$

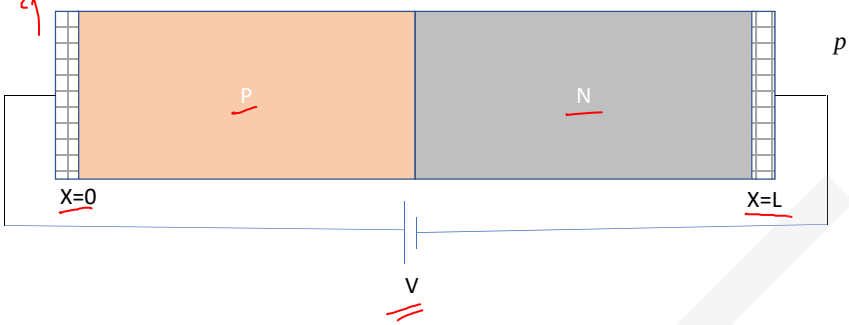
$$\vec{J}_p = -qp\mu_p\frac{\partial \psi}{\partial x} - qD_p\frac{\partial p}{\partial x}$$

$$R = R_n = R_p = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} = \frac{np - n_i^2}{n + p + 2n_i\cosh\left(\frac{E_T - E_i}{k_B T}\right)}\frac{1}{\tau}$$

PROBLEM

Consider a P-N junctions with uniform cross-section so that the problem is one dimensional

Let voltage V is applied across the two ends.



$$n = n_i \exp\left(\frac{E_F - E_I}{k_B T}\right) = n_i . \exp\left(\frac{q.(\psi - \varphi_n)}{k_B T}\right)$$

$$p = n_i . \exp\left(\frac{E_I - E_F}{k_B T}\right) = n_i . \exp\left(\frac{q.(\varphi_p - \psi)}{k_B T}\right)$$

Unknowns:

n, p, ψ

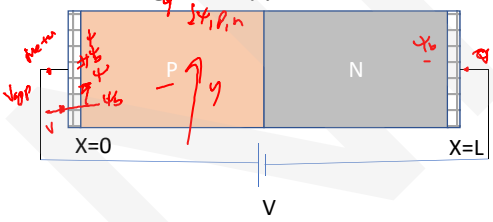
OR

Ψ, φ_n, φ_p

BOUNDARY CONDITIONS

Consider a P-N junctions with uniform cross-section so that the problem is one dimensional

Let voltage V is applied across the two ends.



$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}\left(p - n + C\right),\ C = N_D^+ - N_A^-$$

Ohmic contact at x=0 and x=L ➔ space charge should be zero

- Physical boundary: contact and interface to insulator
  - Ohmic-no potential drop $V_{app} = \psi - \psi_{built-in}$
  - Schottky $V_{app} = \psi - \psi_{built-in} + \psi_{Schottky_Barrier}$
  - Interface to Insulating material

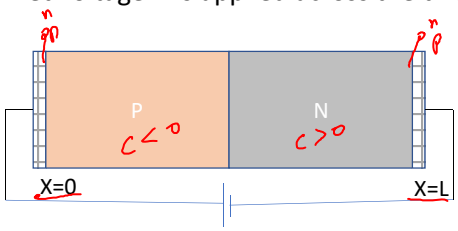
$$\varepsilon_{sc} \frac{\partial \psi}{\partial \vec{n}}\bigg|_{sc} - \varepsilon_{ins} \frac{\partial \psi}{\partial \vec{n}}\bigg|_{ins} = Q_{int}$$

- Artificial boundaries
  - To separate neighboring devices
  - To simplify numerical solution

$$\frac{\partial \psi}{\partial \vec{n}} = 0 \qquad \frac{\partial n}{\partial \vec{n}} = 0 \qquad \frac{\partial p}{\partial \vec{n}} = 0$$

BOUNDARY CONDITIONS

Consider a P-N junctions with uniform cross-section so that the problem is one dimensional
Let voltage V is applied across the two ends.



$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}(p - n + C),\ C = N_D^+ - N_A^-$$

Ohmic contact at x=0 and x=L ➔ space charge should be zero

$$C(0) + p(0) - n(0) = 0 \qquad C(L) + p(L) - n(L) = 0$$

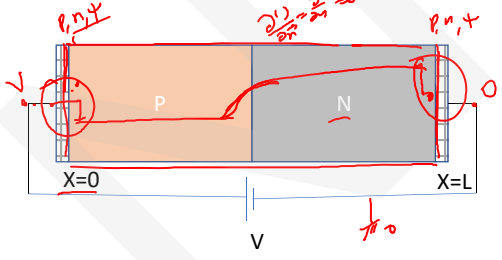
$$p(0) \times n(0) = n_i^2 \qquad p(L) \times n(L) = n_i^2$$

$$p(0) = \frac{-C \pm \sqrt{C^2 + 4n_i^2}}{2} \qquad n(L) = \frac{C \pm \sqrt{C^2 + 4n_i^2}}{2}$$

$$n(0) = \frac{n_i^2}{p(0)} \qquad p(L) = \frac{n_i^2}{n(L)}$$

BOUNDARY CONDITIONS

Consider a P-N junctions with uniform cross-section so that the problem is one dimensional
Let voltage V is applied across the two ends.



$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}(p - n + C),\ C = N_D^+ - N_A^-$$

Ohmic contact at x=0 and x=L ➔ space charge should be zero

$$C(0) + p(0) - n(0) = 0 \qquad C(L) + p(L) - n(L) = 0$$

$$p(0) \times n(0) = n_i^2 \qquad p(L) \times n(L) = n_i^2$$

$$\psi(0) = V - \frac{k_B T}{q} \ln\left[\frac{p(0)}{n_i}\right] \qquad \psi(L) = 0 + \frac{k_B T}{q} \ln\left[\frac{n(L)}{n_i}\right]$$

$$E_F = -q\varphi_n(0) = -q\varphi_p(0) = -qV \qquad E_F = -q\varphi_n(L) = -q\varphi_p(L) = -q.0$$

$$V_{app} = \psi - \psi_{built-in}$$

$$\psi_{built-in} = \psi - \varphi_n \ \text{ or } \ \psi - \varphi_p$$

DISCRETIZATION AND SCALING

Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q}{\varepsilon}\left(p - n + N_D^+ - N_A^-\right)$$

$$n = n_i.\exp\left(\frac{E_F - E_I}{k_B T}\right) = n_i.\exp\left(\frac{q.(\psi - \varphi_n)}{k_B T}\right)$$

$$p = n_i.\exp\left(\frac{E_I - E_F}{k_B T}\right) = n_i.\exp\left(\frac{q.(\varphi_p - \psi)}{k_B T}\right)$$

Discretization

$$\frac{-\varepsilon}{n_i q}\frac{\psi(x_{i-1}) - 2\psi(x_i) + \psi(x_{i+1})}{(h_x)^2} - \exp\left(\frac{q.(\varphi_p(x_i) - \psi(x_i))}{k_B T}\right) + \exp\left(\frac{q.(\psi(x_i) - \varphi_n(x_i))}{k_B T}\right) - \frac{C(x_i)}{n_i} = 0$$

Scaling

$$\lambda^2 \frac{\tilde{\psi}(i-1) - 2\tilde{\psi}(i) + \tilde{\psi}(i+1)}{(\tilde{h}_x)^2} - -\exp(\tilde{\varphi}_p(i) - \tilde{\psi}(i)) + \exp(\tilde{\psi}(i) - \tilde{\varphi}_n(i)) - \tilde{C}(i) = 0$$

DISCRETIZE EQUATIONS

current density equations

$$\vec{J}_n = -qn\mu_n\frac{\partial\psi}{\partial x} + qD_n\frac{\partial n}{\partial x}$$

$$\vec{J}_p = -qp\mu_p\frac{\partial\psi}{\partial x} - qD_p\frac{\partial p}{\partial x}$$

$$\vec{J}_p(i+1/2) = -q\frac{p(i)+p(i+1)}{2}\mu_p(i+1/2)\frac{\psi(i+1)-\psi(i)}{h} - qD_p(i+1/2)\frac{p(i+1)-p(i)}{h}$$

$$= -\frac{q\mu_p(i+1/2)}{h}\frac{p(i)+p(i+1)}{2}[\psi(i+1)-\psi(i)] - \frac{q\mu_p(i+1/2)}{\theta h}[p(i+1)-p(i)]$$

$$\vec{J}_p(i-1/2) = -\frac{q\mu_p(i-1/2)}{h}\frac{p(i-1)+p(i)}{2}[\psi(i)-\psi(i-1)] - \frac{q\mu_p(i-1/2)}{\theta h}[p(i)-p(i-1)]$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

$$-\frac{\mu_p(M-1)}{2h(M-1)}\cdot\left[\frac{2}{\theta} + \psi(N-1) - \psi(N)\right]\cdot p(N-1) +$$

$$\left\{\frac{\mu_p(M-1)}{2h(M-1)}\cdot\left[\frac{2}{\theta} - \psi(N-1) + \psi(N)\right] \ldots\right.$$

$$\left.+\frac{\mu_p(M)}{2h(M)}\cdot\left[\frac{2}{\theta} + \psi(N) - \psi(N+1)\right]\right\}\cdot p(N)$$

$$+\frac{\mu_p(M)}{2h(M)}\cdot\left[\psi(N) - \psi(N+1) - \frac{2}{\theta}\right]\cdot p(N+1)'$$

$$= h'(N)[G(N) - U(N)]$$

DISCRETIZE EQUATIONS

current density equations- Scharfetter-Gummel Discretization

$$\vec{J}_n = -qn\mu_n \frac{\partial \psi}{\partial x} + qD_n \frac{\partial n}{\partial x}$$

$$\vec{J}_p = -qp\mu_p \frac{\partial \psi}{\partial x} - qD_p \frac{\partial p}{\partial x}$$

$$J^n_{i+1/2} = \frac{eD^n_{i+1/2}}{\Delta}[n_{i+1}B(\frac{\Psi_{i+1}-\Psi_i}{V_t}) - n_i B(\frac{\Psi_i - \Psi_{i+1}}{V_t})]$$

$$B(x) = \frac{x}{e^x - 1}$$

$$J_n(i+1/2) = \frac{q}{h_x}[A_1 n(i+1) + A_2 n(i)]$$

$$A_1 = D(i+1/2)B(\frac{\psi(i+1)-\psi(i)}{k_B T/q}) = \mu_p(i+1/2).\frac{\psi(i+1)-\psi(i)}{\exp(q(\psi(i+1)-\psi(i))/k_B T)-1}$$

$$A_2 = D(i+1/2)B(\frac{\psi(i)-\psi(i+1)}{k_B T/q}) = \mu_p(i+1/2).\frac{\psi(i)-\psi(i+1)}{\exp(q(\psi(i)-\psi(i+1))/k_B T)-1}$$

DISCRETIZE EQUATIONS

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

$$n = n_i.\exp(\frac{E_F - E_I}{k_B T}) = n_i.\exp(\frac{q.(\psi - \varphi_n)}{k_B T}) = n_i.\exp(\frac{V}{V_T})$$

$$p = n_i.\exp(\frac{E_I - E_F}{k_B T}) = n_i.\exp(\frac{q.(\varphi_p - \psi)}{k_B T}) = n_i.\exp(-\frac{V}{V_T})$$

$$\frac{1}{q}\frac{J_p(i+1/2)-J_p(i-1/2)}{h(i)} - G(i) + R(i) = 0$$

$$\frac{1}{q}\frac{J_n(i+1/2)-J_n(i-1/2)}{h(i)} + G(i) - R(i) = 0$$

$$\frac{D^n_{i+1/2}}{\Delta^2}B(\frac{\Psi_{i+1}-\Psi_i}{V_t})n_{i+1} - n_i[\frac{D^n_{i-1/2}}{\Delta^2}B(\frac{\Psi_i-\Psi_{i-1}}{V_t}) + \frac{D^n_{i+1/2}}{\Delta^2}B(\frac{\Psi_i-\Psi_{i+1}}{V_t})]$$
$$+\frac{D^n_{i-1/2}}{\Delta^2}B(\frac{\Psi_{i-1}-\Psi_i}{V_t})n_{i-1} = G_i$$

$$\frac{D^p_{i-1/2}}{\Delta^2}B(\frac{\Psi_i-\Psi_{i-1}}{V_t})p_{i-1} - p_i[\frac{D^p_{i-1/2}}{\Delta^2}B(\frac{\Psi_{i-1}-\Psi_i}{V_t}) + \frac{D^p_{i+1/2}}{\Delta^2}B(\frac{\Psi_{i+1}-\Psi_i}{V_t})]$$
$$+\frac{D^p_{i+1/2}}{\Delta^2}B(\frac{\Psi_i-\Psi_{i+1}}{V_t})p_{i+1} = G_i$$

$$B(x) = \frac{x}{e^x - 1}$$

NUMERICAL SOLUTION APPROACH

- Poisson solvers:
  - Direct
    - Gaussian Elimination
    - LU decomposition
  - Iterative
    - Mesh Relaxation Methods
      - Jacobi, Gauss-Seidel, Successive over-Relaxation
    - Advanced Iterative Solvers
      - ILU, Stone's strongly implicit method, Conjugate gradient methods and Multigrid methods
- Solution Procedures:
  - **Gummel's Approach** – when the constitutive equations are weekly coupled
  - **Newton's Method** – when the constitutive equations are strongly coupled
  - **Gummel/Newton** – more efficient approach

CONCLUSION

- Discussed technique to solving DD equations
- discretization of the semiconductor equations yields a large system of nonlinear algebraic equations
- In case of DD there are three equations (one Poisson and two continuity equations)

Thank you
SEMICONDUCTOR DEVICE MODELING AND SIMULATION

NPTEL ONLINE CERTIFICATION COURSE
SEMICONDUCTOR DEVICE MODELING AND SIMULATION
PROF. VIVEK DIXIT
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology Kharagpur
SOLUTION TO NONLINEAR ALGEBRAIC EQUATIONS
LECTURE: 47

L47 NONLINEAR ALGEBRAIC EQUATIONS

Consider set of three equations

$$F(w) = F(\psi, n, p) = \begin{pmatrix} F_1(\psi, n, p) \\ F_2(\psi, n, p) \\ F_3(\psi, n, p) \end{pmatrix}$$

Set of unknowns

$$w = \begin{pmatrix} \psi \\ n \\ p \end{pmatrix}$$

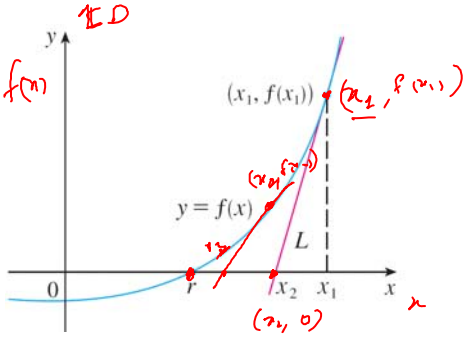
only iterative methods are applicable for the solution of systems of nonlinear algebraic equations

ITERATIVE (NEWTON) METHODS

Let us solve algebraic equation $f(x)=0$ with initial guess x^0

Let us $x^{(1)}$ is k^{th} estimate.
$X^{(2)}$ is K+1st estimate, can be evaluated as

$$x^{(2)} = x^{(1)} - \frac{f(x^{(1)})}{\partial f(x^{(1)})/\partial x}$$



ITERATIVE (NEWTON) METHODS

Let us solve algebraic equation ***f***(x)=0 with initial guess x^0

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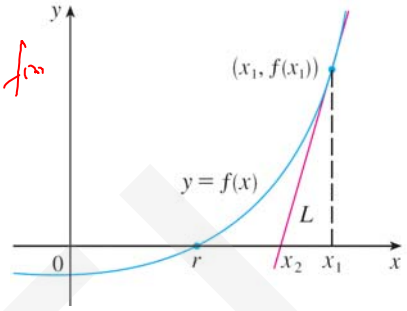
$$x^{(2)} = x^{(1)} - \frac{f(x^{(1)})}{\partial f(x^{(1)})/\partial x}$$



The idea behind Newton's method is that the tangent line is close to the curve and so its x-intercept is close to the x-intercept of the curve (r = root)

$$\delta x^{(k)} = \frac{f(x^{(k)})}{\partial f(x^{(k)})/\partial x}$$

$$\partial f(x^{(k)})/\partial x \,.\, \delta x^{(k)} = f(x^{(k)})$$

ITERATIVE (NEWTON) METHOD THEORY

Assume that a solution w* exists for the system of equations and there exists a neighbourhood of w* within which no other solution exists

All iterative methods are based on a fixpoint equation

$$w = M(w)$$

fixpoint equation is used directly for iteration

$$w^{(k+1)} = M(w^{(k)})$$

$$w^{(k+1)} = M(w^{(k)}) = w^{(k)} - B(w^{(k)})^{-1}.F(w^{(k)})$$

$$M'(w) = I - (B(w)^{-1})'.F(w) - B(w)^{-1}.F'(w)$$

requirements on B(w) such that the iterative scheme converges. variety of operators B(w) exists to satisfy the equation

$$M'(w^*) = I - B(w^*)^{-1}.F'(w^*)$$

The classical Newton method is defined by

$$B(w^*) = F'(w^*)$$

ITERATIVE (NEWTON) METHODS

Initial guess $w^0 = \begin{pmatrix} \psi^0 \\ n^0 \\ p^0 \end{pmatrix}$

Correction for k[th] iteration $\delta w^k = w^{k+1} - w^k = \begin{pmatrix} \psi^{k+1} \\ n^{k+1} \\ p^{k+1} \end{pmatrix} - \begin{pmatrix} \psi^k \\ n^k \\ p^k \end{pmatrix}$

Extending idea to higher dimensions
Newton's method at the k[th] step

$$\begin{pmatrix} \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial p} \\ \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial p} \\ \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial p} \end{pmatrix} \cdot \begin{pmatrix} \delta\psi^k \\ \delta n^k \\ \delta p^k \end{pmatrix} = - \begin{pmatrix} F_1(\psi^k, n^k, p^k) \\ F_2(\psi^k, n^k, p^k) \\ F_3(\psi^k, n^k, p^k) \end{pmatrix}$$

BLOCK-NEWTON METHOD

block iteration scheme (iteration index m) for the solution of the k-th Newton step

$$\begin{pmatrix} \frac{\partial F_1}{\partial \psi} & 0 & 0 \\ \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial n} & 0 \\ \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial p} \end{pmatrix}^k \cdot \begin{pmatrix} \delta\psi^k \\ \delta n^k \\ \delta p^k \end{pmatrix}^{m+1} = - \begin{pmatrix} F_1(\psi^k, n^k, p^k) \\ F_2(\psi^k, n^k, p^k) \\ F_3(\psi^k, n^k, p^k) \end{pmatrix} - \begin{pmatrix} 0 & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial p} \\ 0 & 0 & \frac{\partial F_2}{\partial p} \\ 0 & 0 & 0 \end{pmatrix}^k \cdot \begin{pmatrix} \delta\psi^k \\ \delta n^k \\ \delta p^k \end{pmatrix}^m$$

coefficient matrix is block lower triangular, one can decouple the system into three linear systems which **can be solved sequentially**

SOR BLOCK-NEWTON METHOD

block iteration scheme (iteration index m) for the solution of the k-th Newton step

$$\begin{pmatrix} \frac{\partial F_1}{\partial \psi} & 0 & 0 \\ \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial n} & 0 \\ \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial p} \end{pmatrix}^k \cdot \begin{pmatrix} \delta\psi^k \\ \delta n^k \\ \delta p^k \end{pmatrix}^{m+1} = -\omega \begin{pmatrix} F_1(\psi^k, n^k, p^k) \\ F_2(\psi^k, n^k, p^k) \\ F_3(\psi^k, n^k, p^k) \end{pmatrix} - \omega \begin{pmatrix} 0 & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial p} \\ 0 & 0 & \frac{\partial F_2}{\partial p} \\ 0 & 0 & 0 \end{pmatrix}^k \cdot \begin{pmatrix} \delta\psi^k \\ \delta n^k \\ \delta p^k \end{pmatrix}^m$$

ω is relaxation parameter.

$$\frac{\partial F_1}{\partial \psi}^k \delta\psi^{km+1} = -\omega . F_1(\psi^k, n^k + \delta n^{km}, p^k + \delta p^{km})$$

$$\frac{\partial F_2}{\partial n}^k \delta n^{km+1} = -\omega . F_1(\psi^k + \delta\psi^{km+1}, n^k, p^k + \delta p^{km})$$

$$\frac{\partial F_3}{\partial p}^k \delta p^{km+1} = -\omega . F_1(\psi^k + \delta\psi^{km+1}, n^k + \delta n^{km+1}, p^k)$$

DIRECT METHOD

Direct methods are used for sparse matrices. All direct methods for the solution of sparse systems of linear equations are based on variants of Gaussian elimination

$$Ax = b$$

All methods factorization the coefficient matrix A to the form:

$$P.A.Q = L.U$$

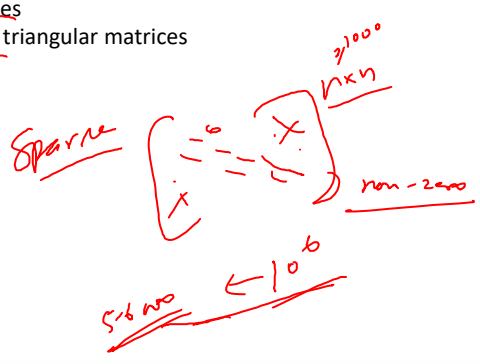
P and Q are permutation matrices

L and U is a lower and an upper triangular matrices

For symmetrical coefficient matrix A :

$$P.A.P^T = L.D.L^T$$

D is a diagonal matrix



GAUSS ELIMINATION-EXAMPLE

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

Matlab Program to solve Ax=b using Gaussian Elimination method

```
clear ; clc ; close all
A = [ 1 2 -1 ; 2 1 -2 ; -3 1 1 ];
b= [ 3 3 -6 ]';
n=3; %size of eqns
x=zeros(n,1);
if det(A) == 0
   disp('Solution does not exist ')
   return
end
M = [ A  b ]
for i = 1:(n-1)
   for j= (i+1) : n
      M(j,:)=M(j,:)-M(i,:)*(M(j,i)/M(i,i));
   end
end
x(n) = M(n,n+1)/M(n,n);
for p = n-1:-1:1
   x(p) = M(p,n+1)/M(p,p)
   for r = p+1:n
      x(p) = x(p)-x(r)*M(p,r)/M(p,p);
   end
end
x
```

```
x =

     3
     1
     2
```

$$M(j,:) = M(j,:) - M(i,:) * M(j,i) / M(i,i)$$

$$x(p) = M(p,n+1)/M(p,p) - \sum_{r=p+1}^{n} x(r) * M(p,r)/M(p,p)$$

CONCLUSION

- Discussed Newton based iterative solving methods and direct method for sparse matrices

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PROBLEM-I

Show that for a degenerate semiconductor relation between the diffusion coefficient D and the mobility μ is given by

$$\frac{D_n}{\mu_n} = \frac{1}{q}\frac{dE_F}{d(\ln n)}$$

This relation reduces to the simpler Einstein relation if the material is nondegenerate, i.e. Boltzmann statistics is used

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

PROBLEM-1

$$\vec{J}_{n,drift} = qn\mu_n E = -n\mu_n \vec{F}_e$$
$$\vec{J}_{n,diff} = qD_n \nabla_r n$$

Equating

$$D_n / \mu_n = -n\vec{F}_e / \left(q\nabla_r n\right)$$

For isothermal solid with parabolic band structure, Force = - dE_F/dx
$D/\mu = (n/q)\, dE_F/dn = (1/q)\, dE_F/d(\ln n)$

For non-degenerate semiconductors, $n = n_i \exp((E_F-E_i)/kT)$
$d(\ln n)/dE_F = 1/kT$
$D/\mu = kT/q$

PROBLEM-2

The Hall effect can be used to determine the type of carriers present in a material. Show that the Hall constant R is given by

$$R = -\frac{1}{qn_i}\frac{\mu_n/\mu_p - 1}{\mu_n/\mu_p + 1}$$

for an intrinsic semiconductor, where n_i is the intrinsic carrier concentration and m_n and m_p are the electron and hole mobilities.

PROBLEM-2: Recall Hall Effect

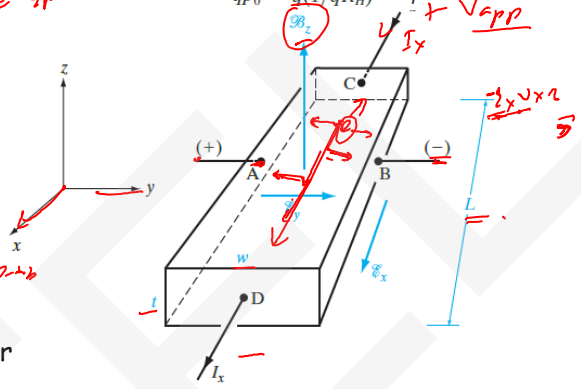
(p-type material)

- **F** = $q(\mathcal{E} + v \times \mathcal{B})$ (Lorentz force)
- Net force in y-axis: $F_y = q(\mathcal{E}_y - v_x\mathcal{B}_z)$
- 0 current in y-direction
- ➔ $\mathcal{E}_y = v_x\mathcal{B}_z$
- $\mathcal{E}_y = (J_x/qp_0)\,\mathcal{B}_z = R_H J_x \mathcal{B}_z$
- Hall voltage
- $V_{AB} = \mathcal{E}_y w$
- Hall coefficient
- $R_H = \mathcal{E}_y/J_x\mathcal{B}_z = 1/qp_0$
  - $V_{AB} < 0$ for n-type
  - R_H and ρ yield majority carrier concentration and mobility

$$R_H = \frac{1}{qp_0} = \frac{J_x\mathcal{B}_z}{q\mathcal{E}_y} = \frac{(I_x/wt)\mathcal{B}_z}{q(V_{AB}/w)} = \frac{I_x\mathcal{B}_z}{qtV_{AB}}$$

$$\rho(\Omega - \text{cm}) = \frac{Rwt}{L} = \frac{V_{CD}/I_x}{L/wt}$$

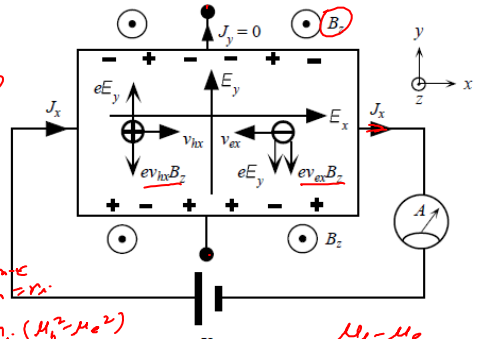
$$\mu_p = \frac{\sigma}{qp_0} = \frac{1/\rho}{q(1/qR_H)} = \frac{R_H}{\rho}$$



PROBLEM-2: Recall Hall Effect

- Hall effect (general)

- $F_{hy}=eE_y-ev_{hx}B_z=ev_{hy}/\mu_h$
- $-F_{ey}=eE_y+ev_{ex}B_z=ev_{ey}/\mu_e$
- $J_y=epv_{hy}+env_{ey}=0$
- $J_x=epv_{hx}+env_{ex}$
- $R_H=E_y/J_xB_z$



$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

- accurate expression including Hall scattering factor r_e and r_h using ($J_y=r_epv_{hy}+r_henv_{ey}=0$),

$$R_H = \frac{r_h p - r_e b^2 n}{e(p + bn)^2},$$

PROBLEM-3: Include Hall Scattering effect

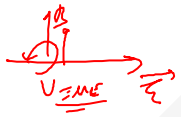
$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

General expression including Hall scattering factor r_e and r_h using ($J_y = r_e e p v_{hy} + r_h e n v_{ey} = 0$),

$$R_H = \frac{r_h p - r_e b^2 n}{e(p + bn)^2},$$

- $r_H = (r_e \sim r_h) = \mu_H / \mu_C = \frac{\langle \tau_m^2 \rangle}{\langle \tau_m \rangle^2}$
- Hall mobility, $\mu_H = r_H \mu$ (~drift mobility)

- For semiconductor with spherical constant-energy surfaces, $\tau_m \propto E^{-1/2}$ for phonon scattering and $\tau_m \propto E^{-3/2}$ for ionized impurity scattering
- $\langle \tau_m^n \rangle = \int_0^\infty \tau_m^n E^{3/2} exp\left(-\frac{E}{kT}\right) dE / \int_0^\infty E^{3/2} exp\left(-\frac{E}{kT}\right) dE$
- $r_H = 3\pi/8 = 1.18$ for phonon scattering and $r_H = 315\pi/512 = 1.93$ for ionized impurity scattering.
- Magnetoresistance effect: under strong magnetic field significant increase in resistivity is observed due to carriers traveling in a path that deviates from the applied electric field.



PROBLEM-4

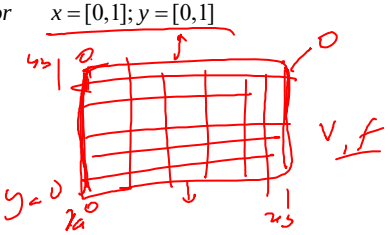
Solve poisson equation 2D : $-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 20\cos(3\pi x) \times \sin(2\pi y)$ *for* $x = [0,1]; y = [0,1]$

Dirichlet boundary (lext and right edge)

$u(x=0) = 0$

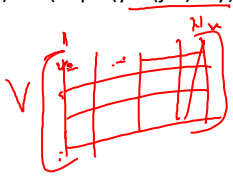
$u(x=1) = 1$

Neumann boundary for y-axis (upper and lower edge) $\frac{\partial u}{\partial \vec{n}} = 0$



```
%--Initialize
V(1:Nx,1:Ny)=0.0;
f(1:Nx,1:Ny)=0.0;
for i=1:Nx,
for j=1:Ny,
f(i,j)=20*cos(3*pi*(xa+(i-1)*dx))*sin(2*pi*(ya+(j-1)*dy));
end
End
% Boundary conditions
V(1,:)=0;
V(Nx,:)=1; %% Dirichlet in x
g=0; %% Neumann in y
```

```
%--Define the 2D mesh
loop=1;
N=20; Nx=N; Ny=N;
xa=0; xb=1; ya=0; yb=1;
dx=(xb-xa)/(Nx-1);
dy=(yb-ya)/(Ny-1);
%% since dx = dy we can use h = dx = dy
h=dx;
M=400; % maximum iteration value
```

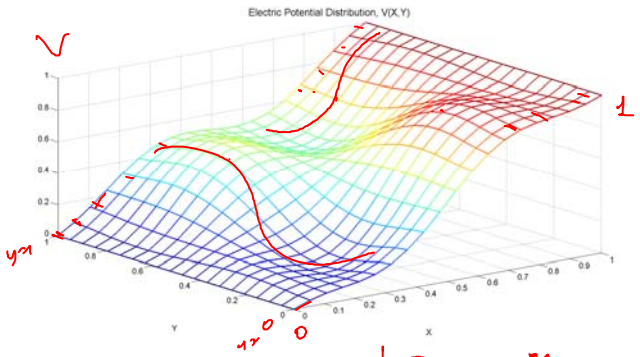


PROBLEM-4

```
w=cos(pi/Nx)+cos(pi/Ny); % Converging Term
Ncount=0;
loop=1;
while loop==1;
  Rmin=0;
j=1;
for i=2:Nx-1
  Residue=w.*(0.25.*(V(i-1,j)+V(i+1,j)+2*V(i,j+1)+2*g*h+(h^2)*f(i,j))-V(i,j));
  Rmin=Rmin+abs(Residue);
  V(i,j)=V(i,j)+Residue;
end
for j=2:Ny-1
for i=2:Nx-1
  Residue=w.*(0.25.*(V(i-1,j)+V(i+1,j)+V(i,j-1)+V(i,j+1)+(h^2)*f(i,j))-V(i,j));
  Rmin=Rmin+abs(Residue);
  V(i,j)=V(i,j)+Residue;
end
end
j=Ny;
for i=2:Nx-1
  Residue=w.*(0.25.*(V(i-1,j)+V(i+1,j)+2*V(i,j-1)+2*g*h+(h^2)*f(i,j))-V(i,j));
  Rmin=Rmin+abs(Residue);
  V(i,j)=V(i,j)+Residue;
end
Rmin=Rmin/(Nx*Ny); % Average Residue per grid point
if(Rmin>=0.00001)
  Ncount=Ncount+1;
  if(Ncount>M)
    loop=0;
    disp(['solution doesnt converge in ',num2str(M),' iter'])
  end
else
  loop=0;
  disp(['solution converges in ',num2str(Ncount) ,' iteration'])
end
end
```


PROBLEM-4

```
% Plot the result
X=xa:dx:xb;%1:Nx;
Y=ya:dy:yb; %1:Ny;
mesh(X,Y,V','linewidth',2)
h=gca;
get(h,'FontSize');
set(h,'FontSize',16);
%colorbar('location','eastoutside','fontsize',12);
%axis([1 Nx 1 Ny])
xlabel('X','fontSize',16);
ylabel('Y','fontSize',16);
title('Electric Potential Distribution, V(X,Y)','fontsize',20);
fh = figure(1);
set(fh, 'color', 'white');
```



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